Adjoint methods for acoustics

Paul Hursky and Michael B. Porter

Science Applications International Corporation

Bruce D. Cornuelle, William S. Hodgkiss, and William A. Kuperman

Scripps Institution of Oceanography

2004 ONR Uncertainty DRI Review, June 15-16.

Work sponsored by ONR code 32.

Outline

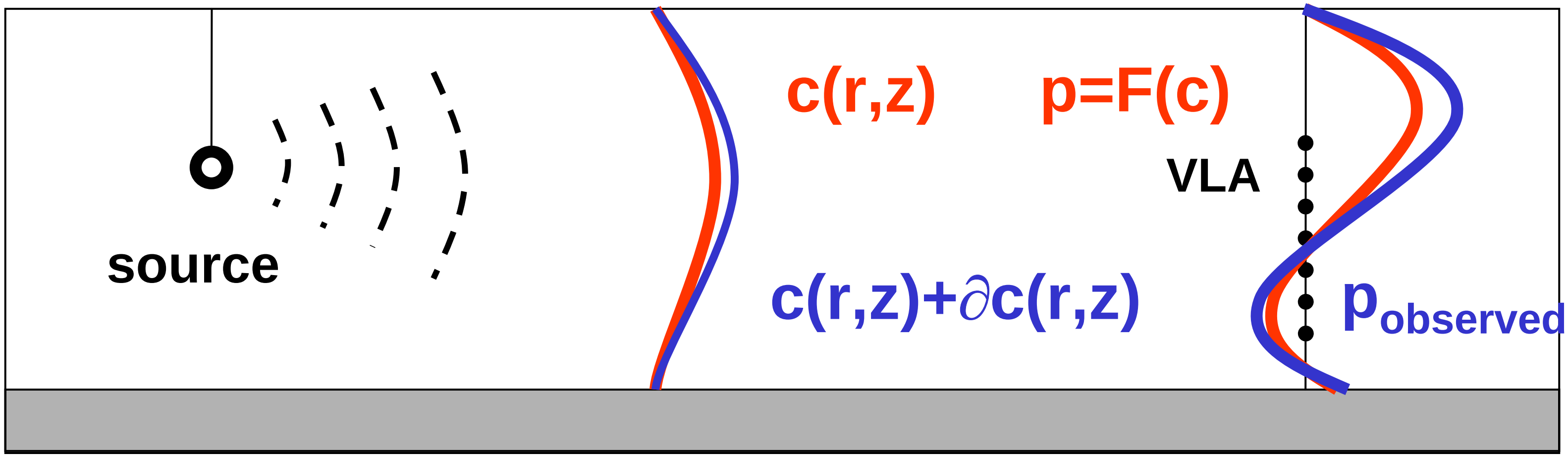
- Adjoint introduction:
  - Iterative steepest descent formulation
  - Imaging range-dependent propagation: solibores and a “bottom inclusion”
  - Adjoint for PE model
- Applications:
  - What part of the environment are my observations sensitive to?
  - Monitoring and tracking internal tides

Adjoint references

- Optimal control theory – Pontryagin principle
- Diffraction tomography (Devaney)
- Geophysical inversion (Tarantola)
- Oceanographic inversion (Munk, Wunsch, Bennett)
- Electromagnetic tomography (Dorn)

- Frechet derivatives for iterative scattering algorithms (Norton)
- Adjoint method for geoacoustic inversion (Asch, Le Gac, Helluy)
- Normal mode adjoint for 3D propagation sensitivity (Aaron Thode)
- Adjoint modeling for acoustic inversion, JASA February 2004 (Hursky, Porter, Cornuelle, Kuperman, Hodgkiss)

Performance prediction errors



Configuration: **source and VLA receiver**

Acoustic model *F*: ***p(c)= F(c)***

Data: $p_{observed} = p(c) + \partial p(p, c, \partial c)$

Solution:

Find c to reduce data-model misfit:

$$\min_c J(c) = || p(c) - p_{observed} ||$$

Recipe for an adjoint model

- ***Forward model*** predicts pressure ***p*** for a given set of medium properties ***c***:

$$\boldsymbol{p}=F(\boldsymbol{c}) \qquad \boldsymbol{p}+\partial\boldsymbol{p}=F(\boldsymbol{c}+\partial\boldsymbol{c})$$

- ***Tangent linear model*** predicts change in pressure $\partial\boldsymbol{p}$ due to changes $\partial\boldsymbol{c}$ in the medium:

$$\partial\boldsymbol{p} = dF(\boldsymbol{p},\boldsymbol{c}, \partial\boldsymbol{c}) = A(\boldsymbol{p},\boldsymbol{c})\, \partial\boldsymbol{c}$$

- ***Adjoint model*** propagates observation errors $\partial\boldsymbol{p}$ back to medium perturbations $\partial\boldsymbol{c}$, calculating gradient for steepest descent:

$$\partial J/\partial\boldsymbol{c} = dF^{*}(\boldsymbol{p},\boldsymbol{c}, \partial\boldsymbol{p}) = A^{*}(\boldsymbol{p},\boldsymbol{c})\, \partial\boldsymbol{p}$$

Adjoint is *Ax=b* steepest descent solution

Can use *pseudo-inverse* of ***A***: $x = A^{+}b$ (if feasible)

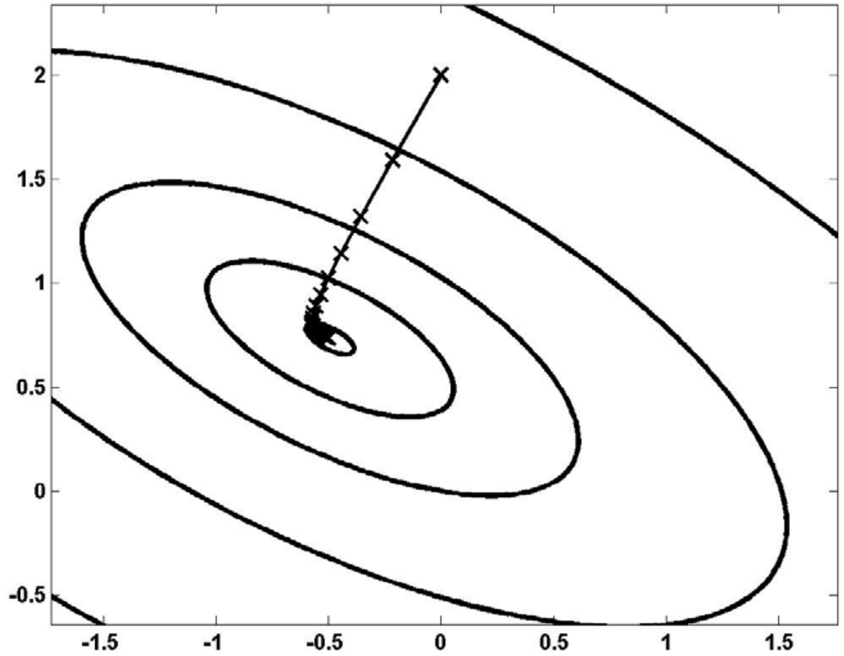
OR, can do *iterative steepest descent*:

$$J(x) = (Ax-b)^{H}(Ax-b)$$

$$\frac{\partial J}{\partial x} = A^{H}(Ax-b)$$

$$x_{i+1} = x_i - \alpha \frac{\partial J}{\partial x}$$

$$x_{i+1} = x_i - \alpha A^{H}(Ax_i - b)$$



Adjoint $\boldsymbol{A^H}$ operates on modeling error ***Ax - b*** to produce gradient of ***J*** with respect to ***x*** (perturbations in c)

PE marching solution as linear system

Tangent linear model propagates pressure from medium perturbation $\boldsymbol{u_n}$ at range $\boldsymbol{n}$ to receiver at range $\boldsymbol{N}$:

$$B_n = F_{N-1}F_{N-2}\ldots F_{n+1}G_n \qquad\qquad p_N = B_n u_n$$

Stack $\boldsymbol{u_n}$ from all ranges to form $\boldsymbol{x}$:

$$x = \begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_N \end{bmatrix} \qquad A = \begin{bmatrix} B_0 & B_1 & \ldots & B_{N-1} \end{bmatrix}$$

$$b = p_N$$

$$Ax = b$$

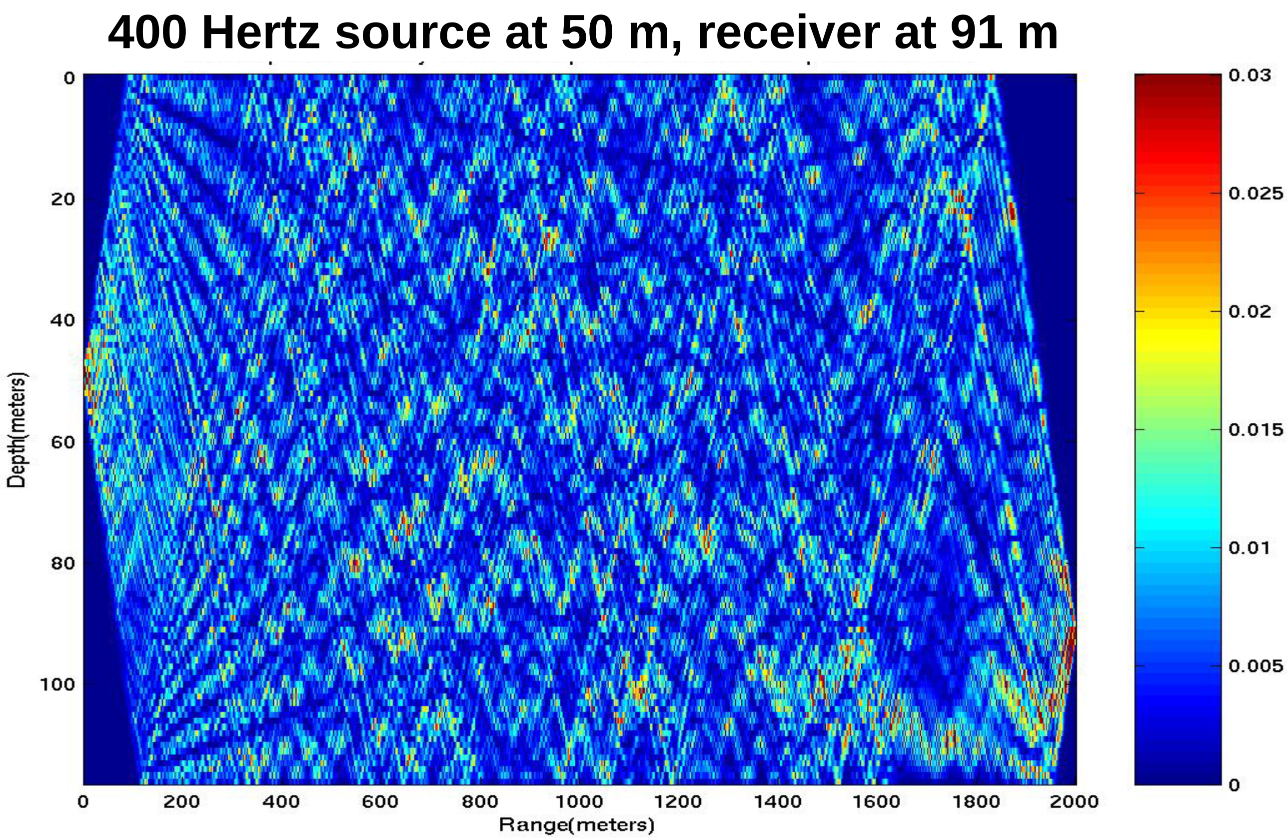
What adjoint of (linearized) PE does

$$A = \begin{bmatrix} B_0 & B_1 & \ldots & B_{N-1} \end{bmatrix} \quad A^H = \begin{bmatrix} B_0{}^H \\ B_1{}^H \\ \vdots \\ B_{N-1}{}^H \end{bmatrix}$$

Adjoint model propagates data-model misfit from receiver at range N to medium perturbation at range n:

$$B_n{}^H = G_n{}^H F_{n+1}{}^H \ldots F_{N-2}{}^H F_{N-1}{}^H$$

Run forward model N times, or adjoint model 1 time for sensitivity



p+∂p=F(c+∂c)	***∂J/∂c = dF*(p,c, ∂p)***
N different ∂c	***1 ∂p***

Iterative process using adjoint

Run forward model:

$$p_{n+1} = F_n p_n + G_n u_n$$

Initialize adjoint model at receiver:

$$\lambda_N = p_N - p_{obs}$$

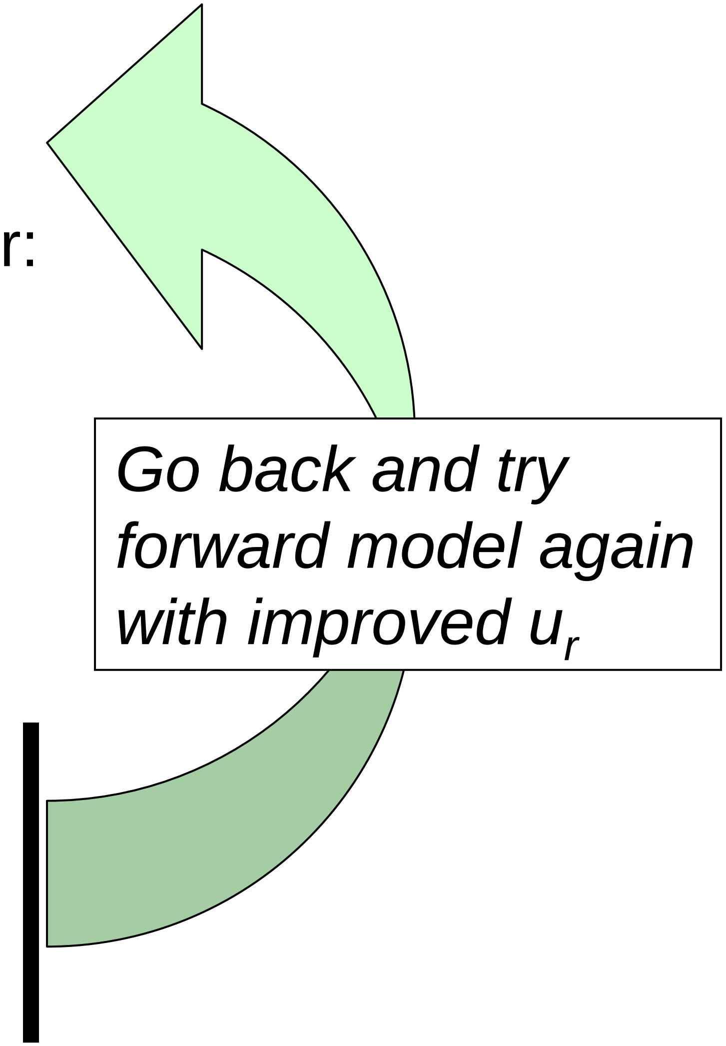
March λ back from receiver

to source via adjoint model:

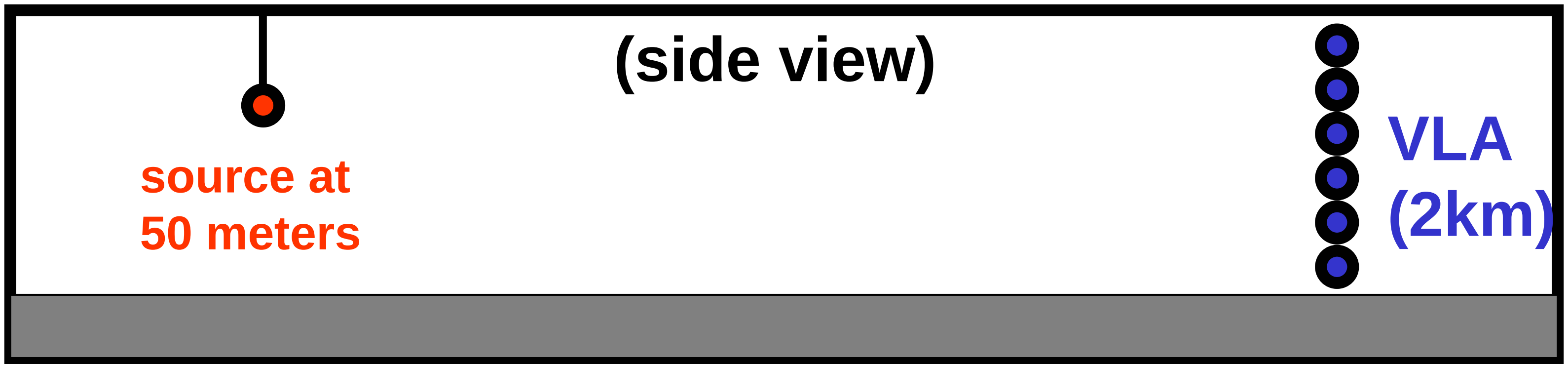
$$\lambda_n = F_n^H \lambda_{n+1}$$

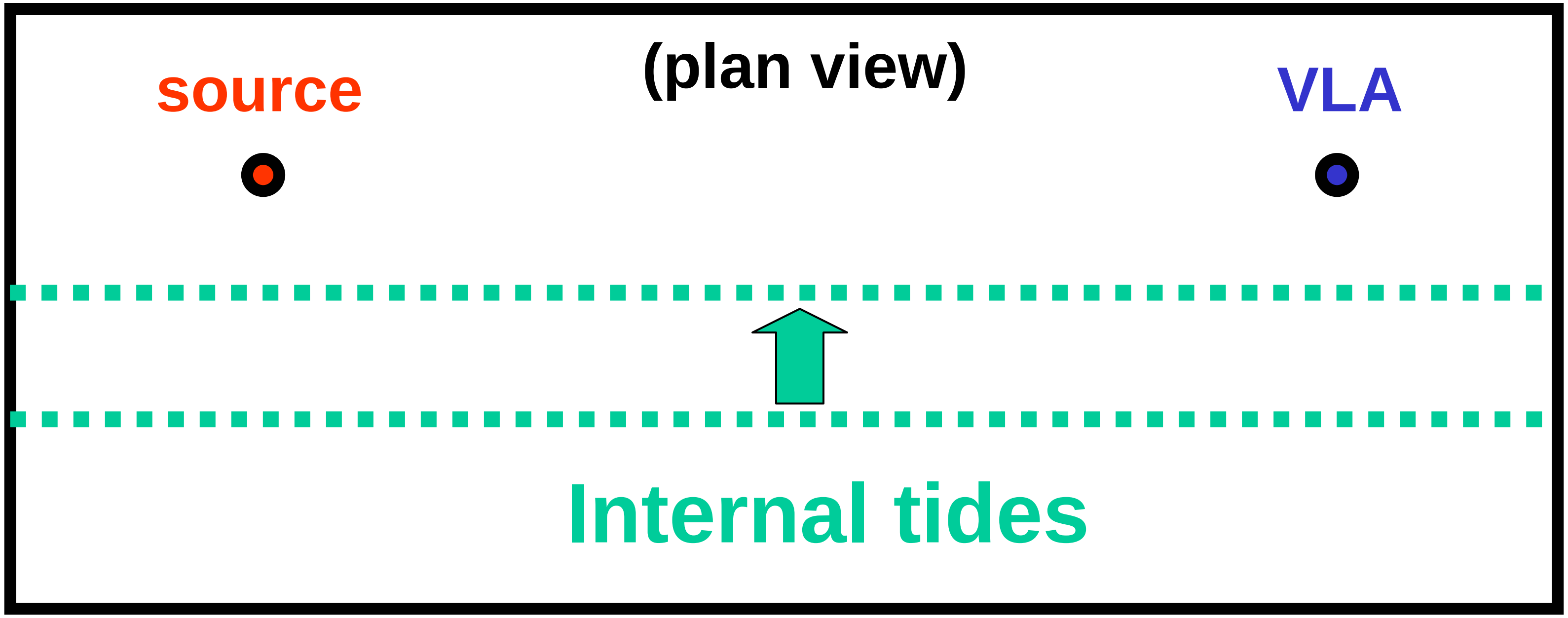
Medium parameters $\boldsymbol{u_n}$ from $\boldsymbol{\lambda_n}$:

$$u_n = -G_n^H \lambda_{n+1}$$

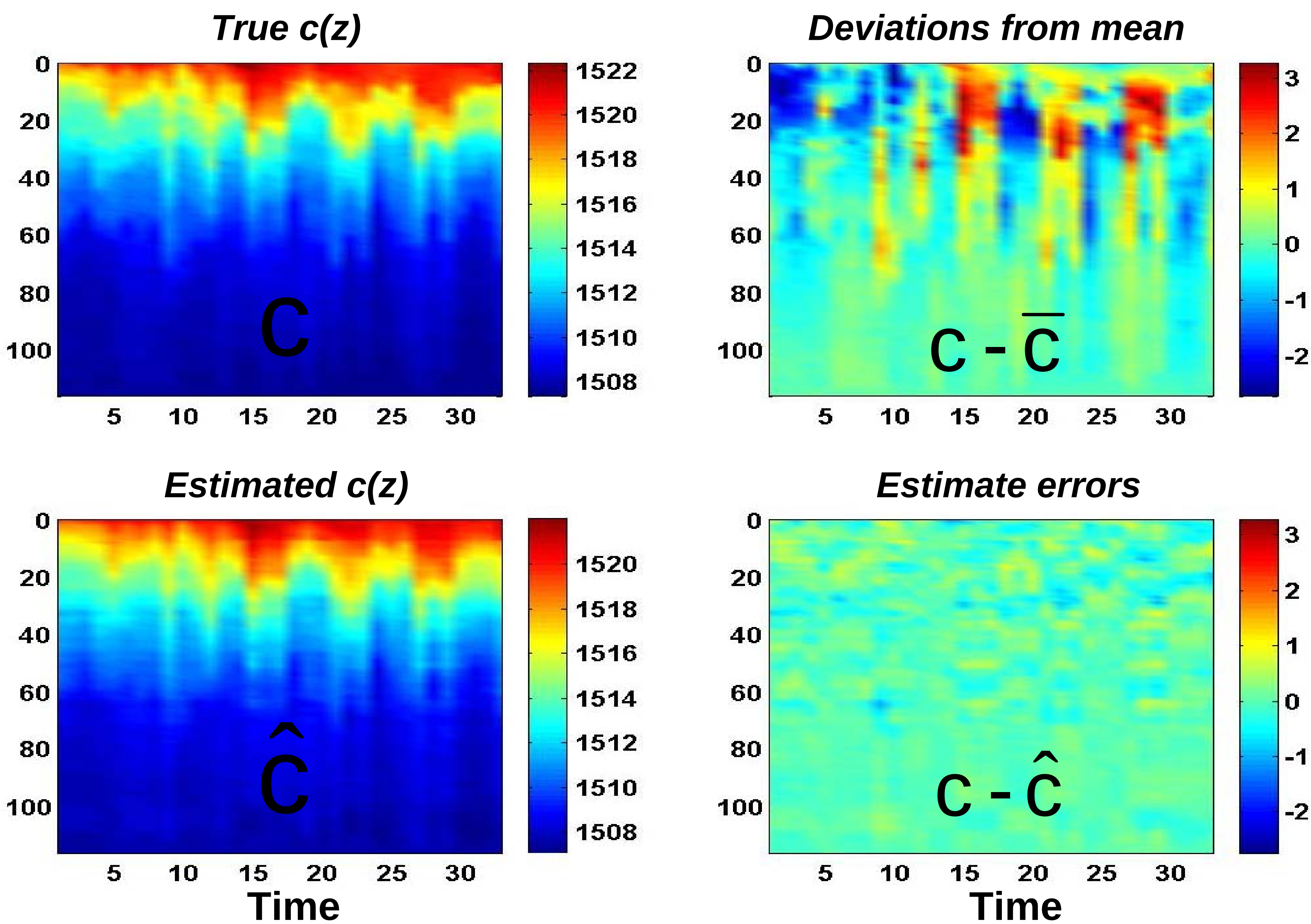


Configuration for monitoring internal tides

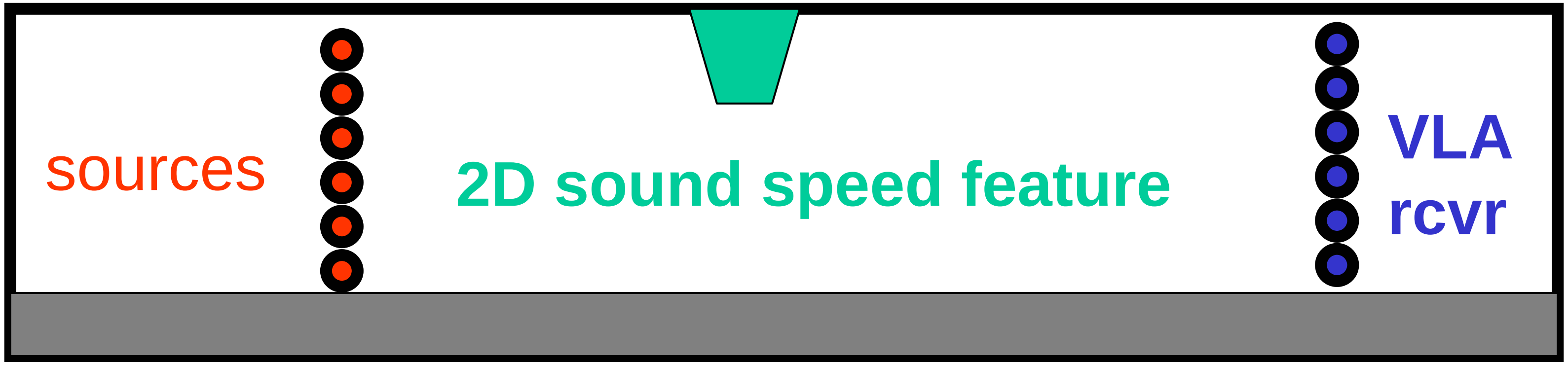




Tracking internal tides



Range-dependent scenario

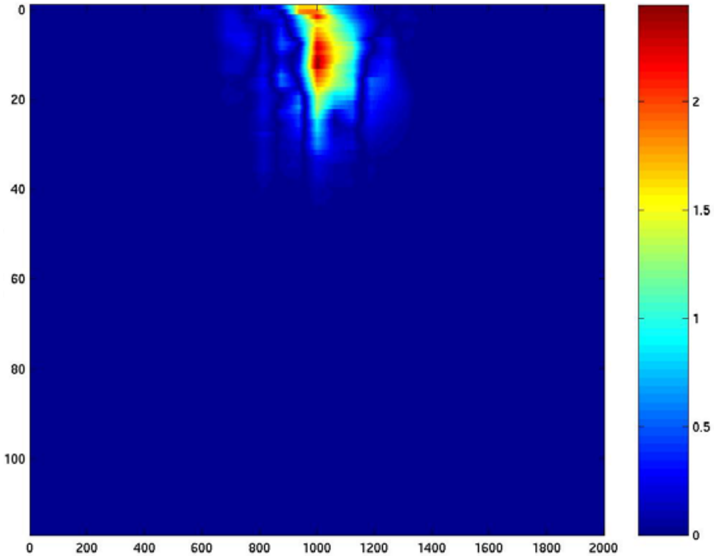


What is messing up performance prediction – a solibore, or a bottom inclusion?

Solibores at three different ranges

Unknown dc

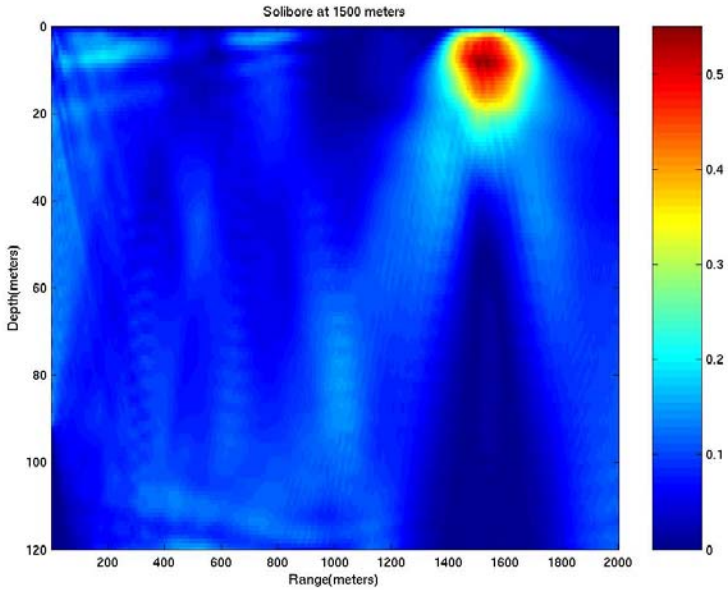
depth



range

Estimated dc

depth

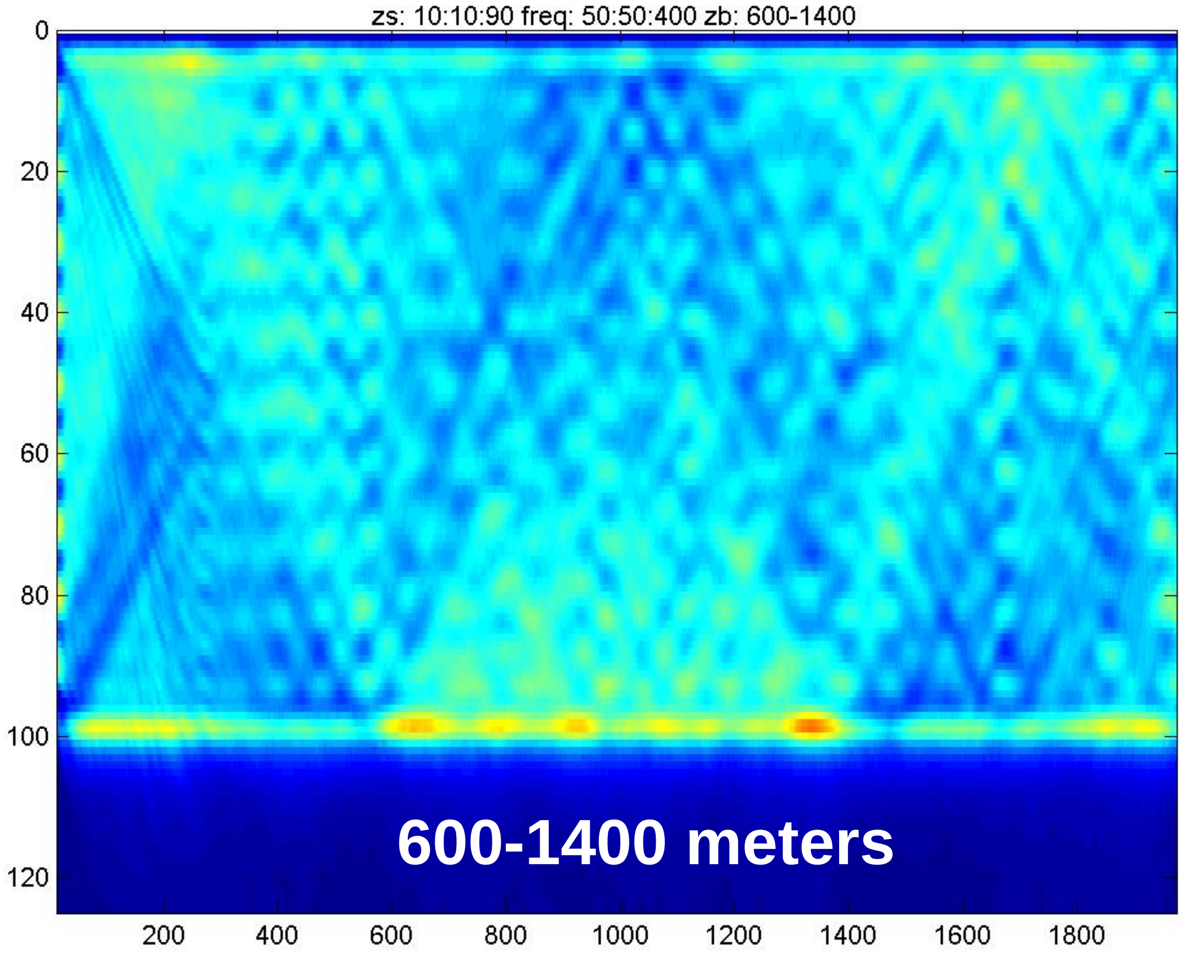


range

Multiple sources and receivers and a wideband source were used to produce this 2D result.

One forward modeling run and one adjoint modeling run (per source) were used.

Bottom inclusions of 3 different lengths



Summary

- Adjoint attractive for calculating sensitivities where it is not possible to use a low-dimensional parameterization
- Adjoint technique is limited to linear regime (solving for small perturbations) and configurations where multiple scattering is not too strong